Calculation of Permeability for Granular Porous Media – 17030

Cheo Kyung Lee and Seang Se Be

Handong Global University
3 Namsong-ri, Heunghae-eub, Buk-gu, Pohang, Kyungbuk, 791-708
Republic of Korea

ABSTRACT

The flow of a viscous fluid is considered to calculate the permeability of a porous medium with granular solid phase. Under the assumption of periodicity a unit cell which contains solid phase of granular shape is selected on the microscale and the flow field is calculated by solving Stokes problem in which inertial effects are ignored. A Boundary-Vale Problem is obtained by applying the theory of homogenization to slow viscous flow through porous structure on the microscale. It is shown that the permeability is dominantly determined by the flow direction along which the pressure gradient on the macroscale is imposed.

INTRODUCTION

When a pressure gradient over a porous medium on the macroscale is imposed externally, the fluid in the pores is driven to flow through the medium as a result of balance between the pressure gradient and the viscous drag exerted by the solid boundary on the fluid. Permeability of the medium is of fundamental importance which characterizes the flow behavior and pattern. For effective management of the underground repository it is essentially important to know the flow characteristics which may lead to the determination (or reliable estimation) of the medium permeability. Knowledge of flow characteristics will also enable one to readily investigate the transport of contaminant released in the medium.

In this study, a medium which is composed of granular solid particles with fluid in the pore space is chosen and the flow field in the pore space is calculated from which the permeability is determined. The calculations are carried out by using FLUENT packed in ANSYS. The theoretical framework is based on the homogenization theory which systematically combines the processes on the microscale and deduces the governing equations and the effective coefficients on the macroscale [1]. The selected microcell geometry of granular shape is especially useful because any contaminant released in a medium spreads out quickly in a granular medium.

Under two basic assumptions, (i) the periodicity of the medium structure on the microscale with periodic length ℓ and (ii) the periodicity of all variables and material properties over the same length. It is noted that the periodicity assumption is not very restrictive because the distributions and arrangements over the periodic length are quite arbitrary.

It is shown that the permeability is dominantly influenced by the direction of external pressure gradient. If the arrangement of solid grains is such the geometry is symmetric about the coordinate planes, the transverse components of the permeability vanish.

It is emphasized that the approach in the present study does not assume any ad hoc or phenomenological assumptions, except the periodicity assumption which is in a sense not restrictive, and starts from the basic governing laws on the microscale and deduces the effective relations on the macroscale systematically by using the multiple-scale perturbation procedure. The permeability is calculated, not estimated, from the solutions to the Stokes problem, a boundary-value problem defined in in the pore space of a microcell.

THE GOVERNING RELATIONS ON THE MICROSCALE

The porous medium is composed of the solid phase (Ωs) and the fluid phase (Ω_f) which is saturated by a liquid. The microscale cell domain is represented by $\Omega = \Omega s + \Omega_f$. Each phase is assumed to be connected throughout the porous medium. Fluid flow is induced by a pressure gradient imposed over the medium on the macroscale. Hence, on the microscale, the leading order pressure gradient appears to be constant with microscale correction which is determined by solving a microcell boundary-value problem.

The basic governing relations and the boundary conditions that must be satisfied in the fluid domain Ω_f are desribed without showing the explicit forms..

The governing equations for the fluid on the microscale in the fluid phase (Ω_f) are the conservation laws of mass and momentum[1].

On the boundary Γ between the solid and fluid, the liquid velocity vanishes

The governing equations and the boundary conditions are normalized by using the representative scales. It is assumed the inertial effects are small, i.e., the Reynolds number is small.

MULTIPLE SCALE ANALYSIS

The distinguishing features of the multiple scale perturbation analysis are briefly summarized. In view of the scale disparity of the porous medium, two distinct length scales are introduced: the microscale - the fast scale which is equivalent to the representative elementary volume in the traditional treatment of the process and the macroscale - the scale over which the processes of interest take place from the viewpoint of reservoir engineering and management.

The variables are expanded as perturbation series in the following small parameter

$$\frac{\ell}{\ell'} = \epsilon \ll 1$$
(Eq. 1)

in which ℓ is the microscale length and ℓ' the macroscale length. Expanding the governing equations and boundary conditions, the microscale boundary-value problems are investigated separately according to the respective order of ϵ and, through volume-averaging over the micro-cell, the effective macroscale governign equations are derived.

In the process of the multiple scale analysis, a canonical micro-cell boundary-value problem (in this case the Stokes problem) is defined whose solution is used in the calculation of the effective macroscale coefficients by averaging over the micro-cell volume.

THE BOUNDARY-VALUE PROBLEM IN THE UNIT CELL

For a porous medium with periodic arrays of unit cells a boundary-value problem is defined during the process of applying the multiple-scale expansions to the basic governing relations on the microscale. Specifically the fluid pressure at the leading order is shown to be independent of the microscale. The fluid velocity in the pore and the correction for pressure are then represented in terms of the macroscale pressure gradient as[2]

$$\begin{aligned} \mathbf{v}^{(0)} &= -\mathbf{K}\cdot\nabla' p^{(0)} \\ p^{(1)} &= -\mathbf{S}\cdot\nabla' p^{(0)} \end{aligned}$$
(Eq. 2a, b)

in which dimensionless variables are used and the primed gradient operator is with respect to the macroscale.

For fluid flow the following Stokes problem in dimensionless variables is defined:

$$\begin{aligned} \nabla^2\mathbf{K} - \nabla\mathbf{S} + \mathbf{I} &= 0 \quad \text{in} \quad \Omega_f \\ \nabla\cdot\mathbf{K} &= 0 \quad \text{in} \quad \Omega_f \\ \mathbf{K} &= 0 \quad \text{on} \quad \Gamma \\ \langle\mathbf{S}\rangle &= 0 \\ \mathbf{K} \text{ and } \mathbf{S} \text{ are} \quad & \Omega- \quad \text{periodic.} \end{aligned}$$
(Eq. 3a-e)

In the above, **K=**K_{ij} and **S=**S_j are the fluid velocity in the i-th direction and the fluid pressure variation in the micro-cell due to externally imposed pressure gradient in the j-th direction. The unprimed gradient operator is with respect to the microscale coordinates. The pair of angle brackets in (Eq. 3d) is the volume average over Ω as defined below in (Eq. 4).

Equations (Eq. 3a) and (Eq. 3b) are the momentum conservation of the fluid driven by a unit force with no convective inertia and the continuity equation, respectively. The no-slip condition on the fluid-solid interface is given in (Eq. 3c). Equation (Eq. 3d) is imposed to ensure the uniqueness of the pressure. Lastly (Eq. 3e) is imposed to satisfy the periodicity condition.

The macroscale permeability tensor of rank two is then given by the micro-cell volume average of **K** as

$$\langle \mathbf{K} \rangle = \frac{1}{\Omega} \int_{\Omega} \mathbf{K} d\Omega \tag{Eq. 4}$$

and the Darcy's law is given as

$$\langle \mathbf{v}^{(0)} \rangle = -\langle \mathbf{K} \rangle \cdot \nabla' p^{(0)} \tag{Eq. 5}$$

where the left-hand side is the seepage velocity and the primed gradient is the derivative of the fluid pressure over the macroscale. This serves as the momentum equation on the macroscale.

THE DISCRETIZATION

The geometry of one-fourth of the unit cell on the microscale is as shown in Fig. 1 in which granular solid and fluid phase are shown.

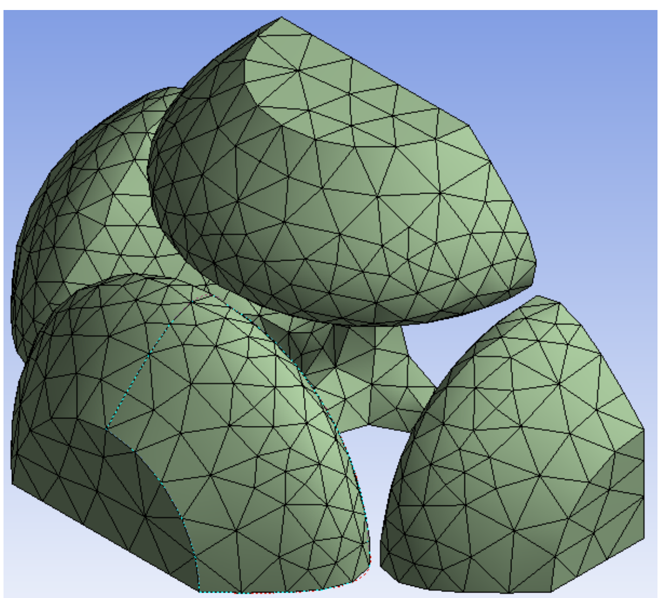
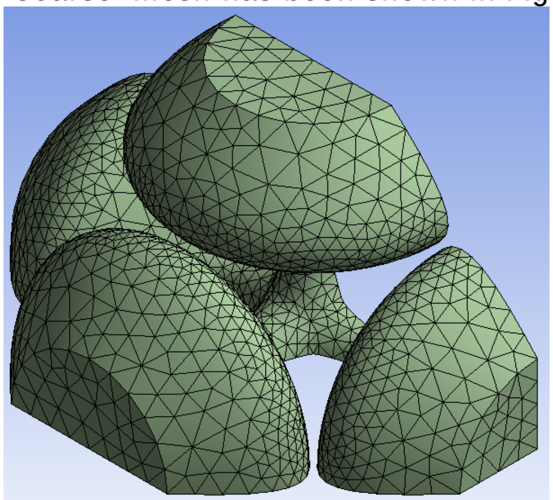


Fig. 1 Geometry of the porous medium with granular solid. Only one-fourth of the microcell is shown.

Except for simple and elementary geometries the Stokes problem does not allow analytic solution and has to be solved numerically. For this purpose commercial software FLUENT for flow analysis (packed together with other ones in ANSYS) has been used.

In solving the Stokes problem, three progressively finer finite element meshes were used. They are labelled as ‘Coarse’, ‘Medium’, and ‘Fine’ respectively. The ‘Coarse’ mesh has been shown in Fig. 1. Two other meshes are shown in Fig. 2.


(a)

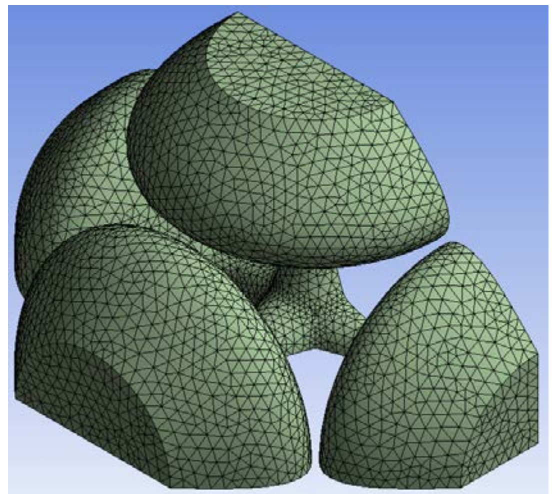

(b)

Fig. 2 Two other meshes used in solving the Stokes problem: (a) Medium and (b) Fine.

THE FLUID VELOCITY AND PRESSURE DISTRIBUTIONS

The velocity distribution in the pore space determined from the Medium mesh, for clarity of the flow field, is shown in Fig. 3.

Due to the external pressure gradient in the x-direction the velocity vestors are aligned predominantly in the x-direction. Notice that, due to the periodicity condition, (Eq. 3e), the velocity arrows on the inlet boundary are repeated on the corresponding portions of the outlet boundary. Notice that, due to symmetry in the geometry, the velocity is nearly zero in the upper part of the pore space which is normal to the x-direction. Hence the upper part of the pore is practically isolated from other part from the flow view point. Due to the no-slip condition for the velocity, (Eq. 3c), the fluid velocity vanishes on the surface of the grains.

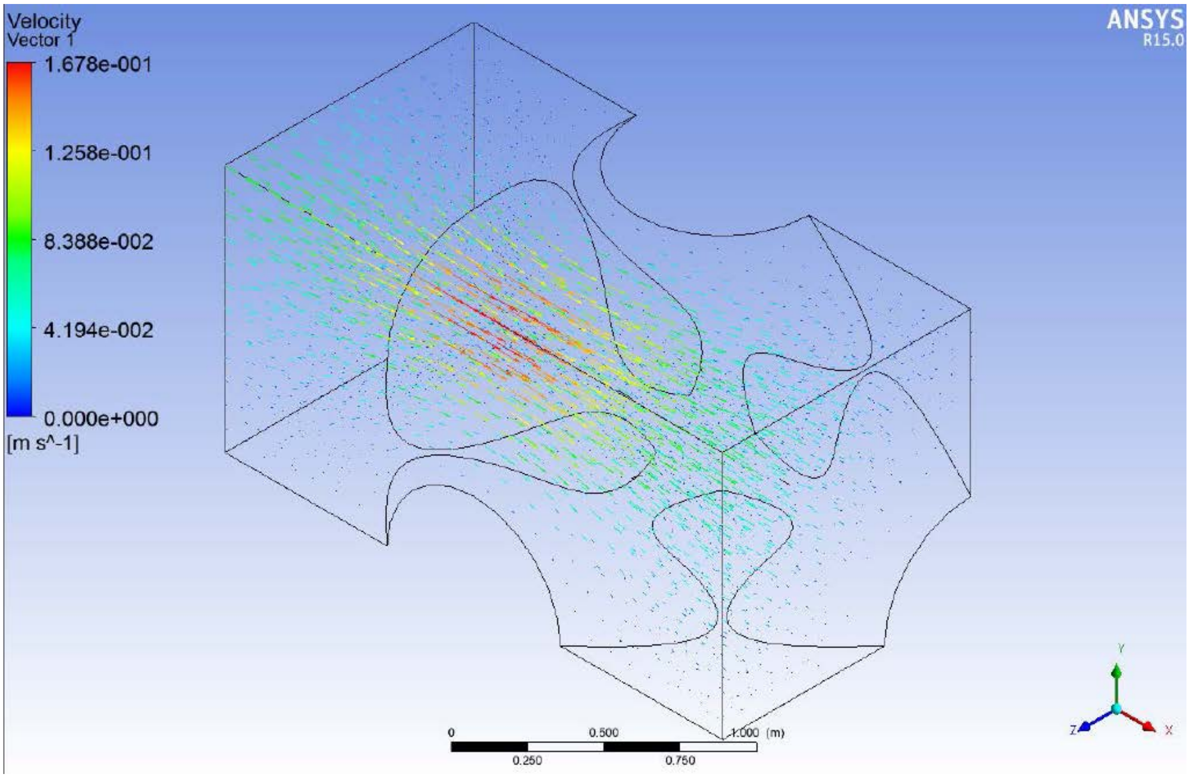


Fig. 3 Velocity distribution in the pores (Medium mesh) when the external pressure gradient is in the x-direction.

The pressure distribution in the pore space is shown in Fig. 4. Notice that the pressure contour lines are quite vertical in the wide pore cross-sections. It changes quite smoothly along the flow channel winding around the solid grains.

Another plot of pressure when viewed from the top (z-axis) is shown in Fig. 5. Again quite smooth variation of the pressure is clearly displayed.

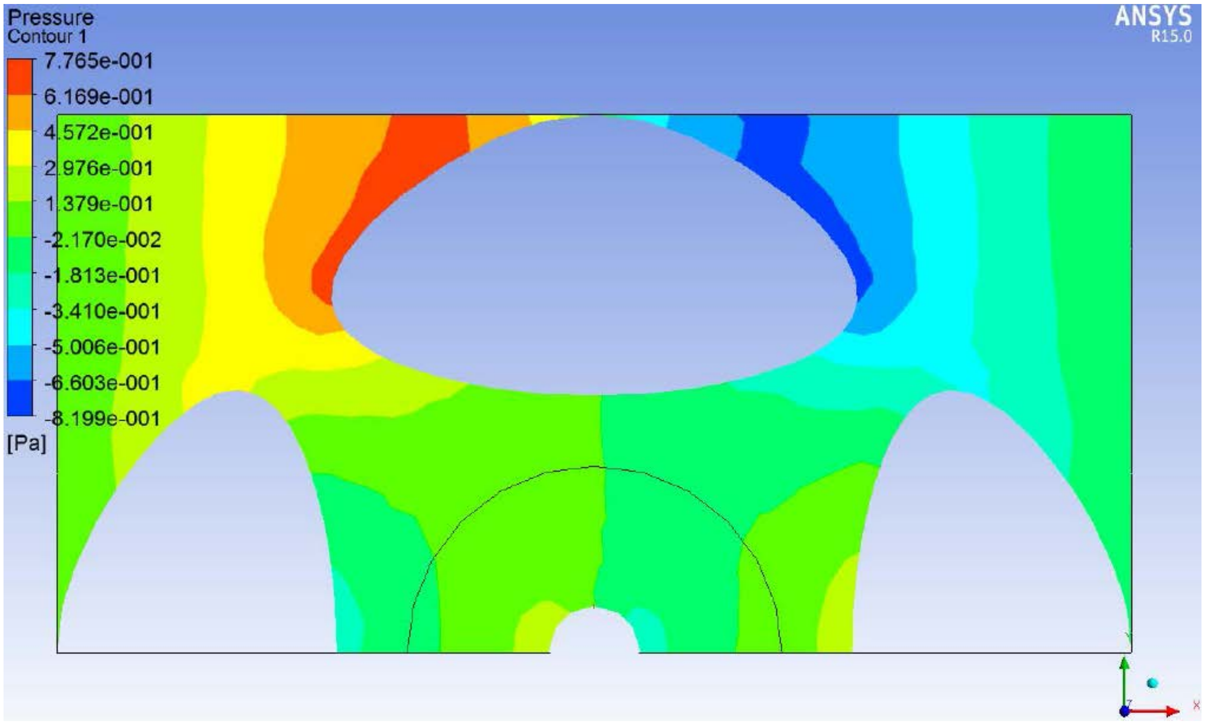


Fig. 4 Pressure distribution in the pores when the external pressure gradient is in the x-direction.

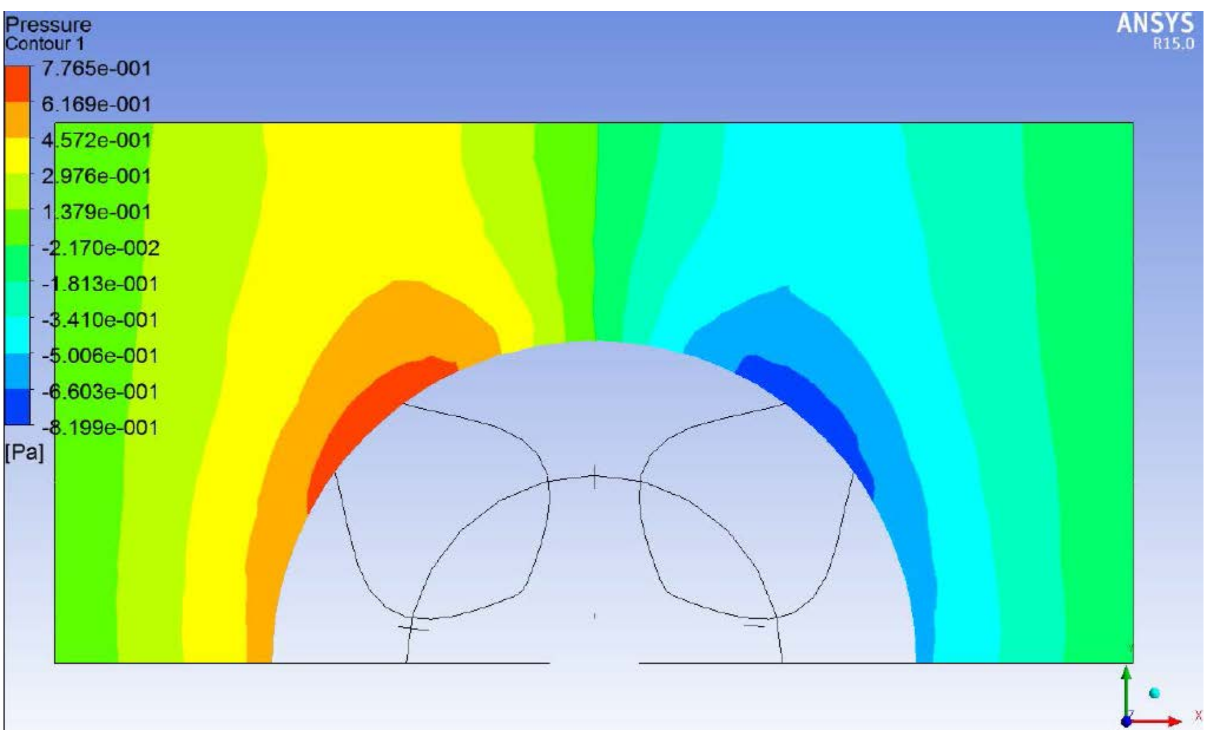


Fig. 5 Pressure distribution in the pores when viewed from the top with the external pressure gradient is in the x-direction.

RESULTS AND DISCUSSION

The permeability is calculated from the velocity field by taking the volume average over the micro-cell using (Eq. 4).

The values calculated from the three different meshes are summarized in Table 1 below.

Mesh	Ωf	Ωs	Ω	Volume Average of Kix		
				i = x	i = y	i = z
Coarse	2.004	1.04	3.042	**2.289E-02**	**1.561E-06**	**3.069E-06**
Medium	1.995	1.05	3.043	**2.245E-02**	**1.243E-06**	**3.592E-07**
Fine	1.991	1.05	3.044	**2.230E-02**	**-7.993E-08**	**1.291E-07**

Table 1 Permeability values from different meshes.

The permeability <Kxx>, the permeability in the x-direction due to the macroscale pressure gradient in the x-direction, converges satisfactorily. The error is less than 0.2% and the result is quite accurate enough. The permeability is dominantly contributed by the flow field in the central wide pore space in Fig. 3.

The volumes of Ωf and Ωs change only a tiny bit. It is the consequence of mesh generation procedure conducted by ANSYS and does not affect the result.

The microcell geometry used in the present study is composed of granular solids and fluid regions. It well represents porous media with round grains such as sands and sandstones.

CONCLUSIONS

From the calculations of the permeability in a porous medium with granular solids in a unit cell on the microscale the following conclusions are drawn.

1. The permeability of a medium with granular particles is dominantly influenced by the pressure gradient on the macroscale.
2. The permeability is clearly from the wide cross-sections in the pore space.
3. The calculation of permeability for a porous medium with general geometry can be carried out efficiently by using numerical methods.

REFERENCES

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2. C.K. LEE, S. Sok, and S. P. YIM, “Calculation of Permeability for Porous Media”, WM2015, Phoenix, March 15-19 (2015).

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